An assessment and enhancement of the newly geopotential models over Egypt territory

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Abstract — Recently, several high and ultra-high degree global harmonic models have been developed. In what follows, An assessment and enhancement of newly Erath Geopotential Models “EGMs” which released between 2015, up to 2017 were done, including corporate data from the gravity satellites CHAllenging Minisatellite Payload “CHAMP”, Gravity Recovery and Climate Experiment “GRACE” and The Gravity field and steady-state Ocean Circulation Explorer Mission “GOCE" dedicated satellite gravimetric mission, with terrestrial gravity anomalies data over the whole area of Egypt. The gravity anomalies which are implied by the models compared with free air gravity anomalies observations while the geoid heights which are implied by the Models compared with discrete geometrical heights from co-located GPS and sprit-leveling. Second step utilize the available GPS/leveling data at discrete points to enhancement the chosen EGMs according to its acting performance “from the first step” all over Egypt territory. The aim of this paper is to improve our knowledge about the performance of the satellite only tracking and combination EGMs which are generated from various satellites or only tracking "CHAMP, GRACE and GOCE" in addition to utilized the available terrestrial data over the Whole area of Egypt to enhance the performance of the chosen EGM from the evaluation step.

The assessment results of the comparison among the different geopotential model with the land gravity data “free air gravity anomalies and GPS\Leveling at discrete points” under investigation in this study have indicated the outstanding performance of EGM [XGM2016] to the other examined GGMs.

EGM [XGM2016] has superior performance with smallest [RMSE] is [20.595 mgal] with respect to [w.r.t] gravity anomaly and [0.577 m] w.r.t geoidal height. An enhancement of EGM [XGM2016] with the GPS\Leveling discrete data points, clarify that the model is enhanced by 2.6% “with using only 80 GPS/leveling data points with bad distribution” than the original one, while the internal accuracy reached to be 44 cm in terms of [R.M.S.E] and 19 cm in terms of stander deviation as shown in table [8] and the external accuracy enhanced by 28%.

Fortunately if there is enough terrestrial data with good distribution over the whole territory this ratio defiantly will be increase.

Keywords— EGM “Earth geopotential models”, Geopotential models, gravity, gravity anomalies, enhancement EGMs, Tailoring EGMs.

1. Introduction

The global and regional modeling of the Earth’s gravity field has been one of the major tasks of geodetic science, geophysics, oceanography and astronomy.

Gravimetric geoid modeling is usually performed using an Earth geopotential model along with a set of detailed local data (Amin, 1983). Therefore, the existence of a high-quality geopotential model that fits the local gravity field is necessary for the determination of an accurate and precise gravimetric local geoid. In geodetic practice, however, the gravimetric approach can be considered as the widely used, most general and flexible trend. Particularly, the least-squares collocation (LSC) algorithm is a vital flexible tool in combining all possible heterogeneous data, pertaining to any irregular data configuration, in one unified solution to solve for any desired type of the anomalous signals.

The higher accuracy of geoid computation required nowadays necessitates the need for an accurate GGM, which in turn necessitates the need for examining the performance of such newly released models in any local area to choose the best of them. Many of such studies have been done before in Egypt such as; (Amin et al., 2002, 2003; Hassouna, 2003).This study aims also to evaluate the behavior of those new models over Egypt to determine which of them is the most appropriate GGM there. In addition to enhance the chosen EGM according to the available GPS/leveling data points.

2. BACKGROUND

The gravity potential, as well as the respective observable quantities, represents a spatial stochastic phenomenon. Hence, the global anomalous gravitational field may be irregular enough to be considered as a realization (or a sample) of a stochastic process (repeated fields), in spite of the fact that there is only one Earth.

the gravitational potential of the Earth, V,

$$V=G\iiint_e \rho(x,y,z)/l.dxdydz, \quad (1)$$

is a harmonic function in outer space (where no attracting masses exist), and there, it satisfies Laplace's equation, which is given as

$$\nabla 2V = \partial 2V/\partial X2 + \partial 2V/\partial Y2 + \partial 2V/\partial Z2 = 0 \quad (2)$$

It follows immediately that applying the Laplacian operator on disturbing potential, T = V-Vn, which is harmonic outside the attracting masses, gives

$$\nabla 2T= \partial 2T/\partial X2 + \partial 2T/\partial Y2 + \partial 2T/\partial Z2 = 0. \quad (3)$$

It can be shown that the Laplacian condition equation [2], for the gravitational potential of the Earth, can be expressed in spherical coordinates as follows (Heiskanen and Moritz, 1967)

$$r2\ \partial 2V/\partial r2 + 2r\ \partial V/\partial r + \partial 2V/\partial \theta 2 + \cot\theta\ \partial V/\partial \theta + (1/\sin 2\theta)\ \partial 2V/\partial \lambda 2 = 0, \quad (4)$$

where r is the geocentric radius, θ is the co-latitude (or the compliment of the geocentric latitude ψ') and λ is the geodetic latitude. The general solution of the above differential equation yields the well-known infinite spherical harmonic expansion of the Earth's gravitational potential, which is expressed by

The spherical harmonic representation of the Earth's gravitational potential,

(Rapp and Pavlis, 1990):- could be

$$V(r,\theta,\lambda) = \frac{GM}{r}\left[1 + \sum_{n-2}^{\infty}\left[\frac{a}{r}\right]^n \sum_{m=-n}^{n} \bar{C}_{nm}^{s}\ \bar{Y}_{nm}(\theta,\lambda)\right] \quad (5)$$

Where:

r is the geocentric distance;

θ is the geocentric co-latitude; and

λ is the longitude;

GM is the geocentric gravitational constant and a is the scaling factor associated with the fully normalized coefficients, Cnm, Ynm

$$\bar{Y}_{nm}(\theta,\lambda) = \bar{P}_{nm}(\cos\theta)\ \cos m\lambda \ \text{ if } m \ge 0$$
$$\bar{Y}_{nm}(\theta,\lambda) = \bar{P}_{nm}(\cos\theta)\ \cos m\lambda \ \text{ if } m < 0 \quad (4)$$

Where:

$P_{nm}[\cos\theta]$ are the fully normalized associated Legendre functions of the first kind (Heiskanen and Moritz, 1967).

The disturbing potential T at a point P [r, θ, λ] is the differences between the actual gravity potential of the Earth and the normal potential of equipotential ellipsoid at P. Based on equation [5] the spherical harmonic representation of T is:

$$T(r,\theta,\lambda) = \frac{GM}{r}\sum_{n-2}^{\infty}\left[\frac{a}{r}\right]^n \sum_{m=-n}^{n} \bar{C}_{nm}^{s}\ \bar{Y}_{nm}(\theta,\lambda) \quad (6)$$

The above formula have been expanded for several numerous processes to get the

Element of the earth's gravity field such as gravity anomalies [Δg] and geoidal height [N]. The relationship between the coefficient of spherical harmonic with gravity anomalies [Δg_{GM}] and geoidal height [N_{GM}] is given by the following formula, respectively:

$$\Delta g_{GM} = \frac{GM}{r^2}\left[\sum_{n-2}^{n_{max}}(n-1)\left[\frac{a}{r}\right]^n \sum_{m=0}^{n} \bar{C}_{nm}^{*}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right]\bar{P}_{nm}(\sin\varphi) \quad (6)$$

$$N_{GM} = \frac{GM}{r\gamma}\left[\sum_{n-2}^{n_{max}}(n-1)\left[\frac{a}{r}\right]^n \sum_{m=0}^{n} \bar{C}_{nm}^{*}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right]\bar{P}_{nm}(\sin\varphi) \quad (7)$$

Where:

GM is the geocentric gravitational constant;

n_{max} is the maximum degree;

n, m is the degree and order;

$\bar{C}_{nm}^{*}$ is the relevant fully normalized spherical harmonic C-coefficients of degree n and order m, reduced for the even zonal harmonics of the WGS-84 reference ellipsoid

$\bar{S}_{nm}$ is the relevant fully normalized spherical harmonic S-coefficients of degree n and order m,

$\bar{P}_{nm}(\sin\varphi)$ is the fully normalized associated Legendre function of degree n and order m,

φ,λ the geocentric latitude and longitude;

γ the normal gravity;

a the scaling factor and r is the geocentric distance.

3. THE AVAILABLE DATA

The gravity anomaly data, Figure [1] shows an irregular distribution with large gaps, especially on land while the coverage of Mediterranean and Red Sea is rather good than the land covering.

The local gravity data used in this study were grouped into two sets as shown in figure [1]. Firstly, all old available free-air gravity anomalies at [800] points, where the sources of these data their number and distribution are well documented in many previous works as shown in (Amin et al., 2002, 2003; Hassouna, 2003) free-air gravity anomaly values at [267] points were obtained from BGI [Bureau Gravimetric International], where their observational mean stander deviation is [0.24mgal], while the stander deviation estimated for older gravity anomaly data distributed all over the whole territory of Egypt is [0.73mgal] on average, secondly Marine free-air gravity anomalies at [31934] points. As can be seen from figure [1], free air gravity data distribution is not homogeneous over the land, with significant gaps, particularly in the eastern and western deserts, while it's approximately homogeneous distributed over the seas. In addition to [100] of known orthometric and ellipsoidal height "geoid undulation" as shown in figure [2].

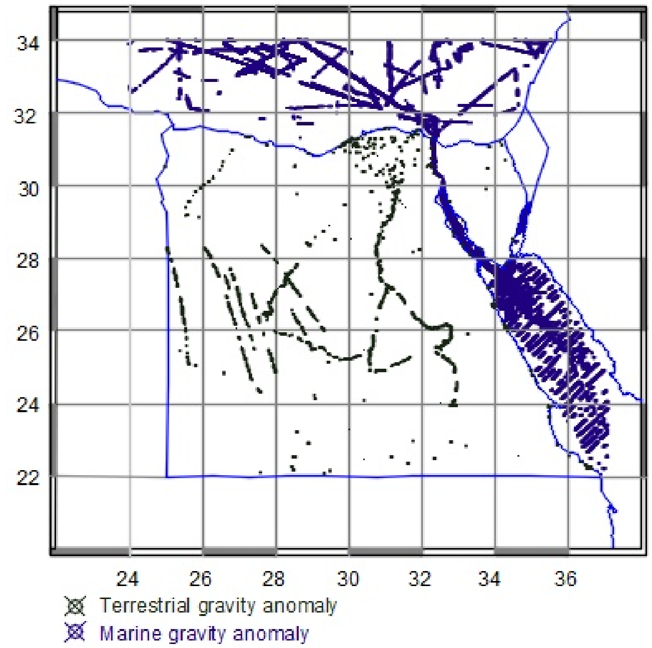

Fig. 1. Free air gravity anomaly distribution over EGYPT

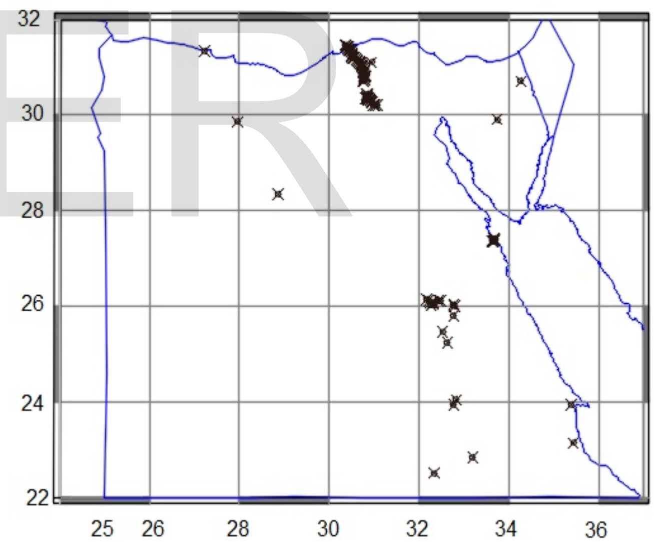

Fig. 2. GPS/Leveling points over 100 scattering point's distribution over EGYPT

TABLE 1

THE AVAILABLE USED DATA

Item	Data No. after filtration
Gravity anomalies [old]	800
Gravity anomalies [BGI]	267
Marine gravity anomalies	31,934
Undulation points	100

4. METHODOLOGY

The statistical analysis preformed into two process the first one is an evaluation the four EGMs to choose one of them to, the second process were the its enhancement the chosen model to the optimum performance utilize the available GPS/leveling which are so poor distributions and few data points available, that step depends on the the [LSC] for all the interpolation grid data from the residual calculation [δN= Nobserved- NXGM 2016] using SAGA software to perform task in the step to add this correction to the EGM XGM2016 [$N_{enhanced} = N_{XGM2016} + \delta N$]

The final process is to check the accuracy according to the 100 GPS/leveling data point were 80 percent using in enhancement and also using to check the internal accuracy upon the rest 20 percent using to check the external accuracy were this percent did not utilize in enhancement.

The precision of the behavior of each model in the case studies here is represented in

Terms of stander deviation [STA.DEV.] of the residual computed from each model as

Follows:

$$\sigma\Delta g_r = \sqrt{\frac{\sum_{i=1}^{n}(\Delta g_{ri} - \overline{\Delta g_r})^2}{n-1}} \quad (7)$$

Where,

$$\Delta g_{ri} = (\Delta g_i - \widehat{\Delta g_i})$$

$$\overline{\Delta g_r} = \frac{1}{n}\sum_{i=1}^{n}\Delta g_{ri}$$

The accuracy of the behavior of each model is represented in terms of root mean square error [R.M.S.E] as follows,

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(model_i - observed_i)^2} \quad (8)$$

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Delta g_i - \widehat{\Delta g_i})^2} \quad (9)$$

TABLE 2

RECENTLY EGMS BETWEEN 2015 UP TO 2017 TO BE EVALUATING IN THIS STUDY

Model	Year	Degree	Data	References
XGM2016	2017	719	A,G, S(GOCO05s)	Pailet al., 2017
NULP-02s	2017	250	S(Goce)	Marchenko et al., 2017
GOCO05c	2016	720	(see model), A, G, S	Fecheret al., 2017
GECO	2015	2190	EGM2008, S(Goce)	Gilardoniet al., 2016

Data: S=Satellite Tracking Data, G = Terrestrial Gravity Data, A = Altimetry Data

TABLE 3

REPRESENTS THE STATISTICS COMPUTATION FOR FREE AIR GRAVITY ANOMALY OVER GRID POINTS [1'X1'] OF THE FOUR MODELS OVER THE WHOLE TERRITORY OF EGYPT.

Free air gravity anomaly	Min mgal	Max mgal	Mean mgal	Std. Dev. mgal	RMSE mgal
GOCO	-165.165	145.631	0.904	29.945	29.959
XGM2016	-166.041	148.706	0.886	29.914	29.927
NULP-02s	-165.573	114.714	0.934	28.468	28.483
GOCO05c	-165.920	145.299	0.863	29.803	29.815

TABLE4

REPRESENTS THE STATISTICS THE COMPARISON AMONG THE TERRESTRIAL GRAVITY ANOMALIES AND THOSE COMPUTED FROM THE DIFFERENT HARMONIC MODELS AT SCATTERING POINTS THREE MODELS OVER THE WHOLE TERRITORY OF EGYPT.

Free air gravity anomaly	Min mgal	Max mgal	Mean mgal	R.M.S. mgal	STD. DEV.Of the residual mgal
$G.A_{terrs}$- XGM2016	-133.272	119.353	-4.129	20.595	20.177
$G.A_{terrs}$- NULP-02s	-192.935	161.272	-5.540	44.424	44.078
$G.A_{terrs}$ - GOCO05c	-134.747	120.088	-5.428	21.975	21.295
$G.A_{terrs}$ - GOCO	-142.302	95.702517	-5.701	22.940	22.221

TABLE 5

REPRESENTS THE STATISTICS THE COMPARISON AMONG THE GPS/LEVELING POINTS AND THOSE COMPUTED FROM THE DIFFERENT HARMONIC MODELS AT SCATTERING POINTS

Geoid undulation	Min m	Max m	Mean m	R.M.S. m	STD. DEV.Of the residual m
N_{obs} - $N_{XGM2016}$	-0.993	0.758	-0.358	0.577	0.451
N_{obs} - $N_{NULP-02s}$	-0.9963	0.889	-0.342	0.620	0.515
N_{obs} - $N_{GOCO05c}$	-0.986	0.767	-0.335	0.569	0.459
N_{obs} - N_{GOCO}	-0.992	0.754	-0.338	0.569	0.460

In table [3] shows the statistical computation of the gravity anomaly derived from different harmonic models over grid [1'x1'] by using SAGA software.

Tables [4] shows the statistics of the comparison among the terrestrial gravity anomalies and those computed from the different harmonic models at scattering points of over whole area of EGYPT by using ordinary kriging technique for interpolation, while table [5] describes the comparison among the terrestrial GPS\Leveling points and those computed from the different harmonic models at scattering points with week distributions over Egypt. The values, of [R.M.S] and [Std. Dev.] shown in columns four and five in tables [3 and 5], are related to the area between 24: 37 E and 22: 34 N which is larger than Egypt territory. The chosen area to be larger than EGYPT territory, therefore to illustrate the performance of those EGMs not only on the land but also on the marine.

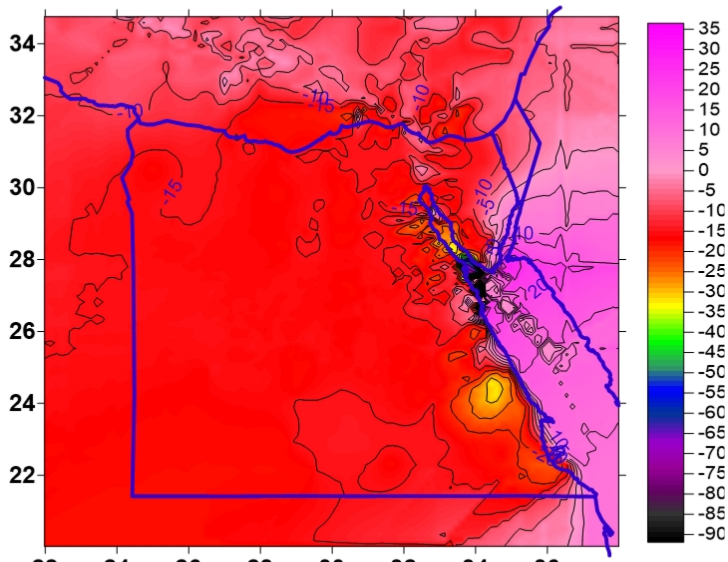

Fig.3 Δg_{fa} residuals referred to EGM XGM2016 with terrestrial data for the whole area of Egypt.

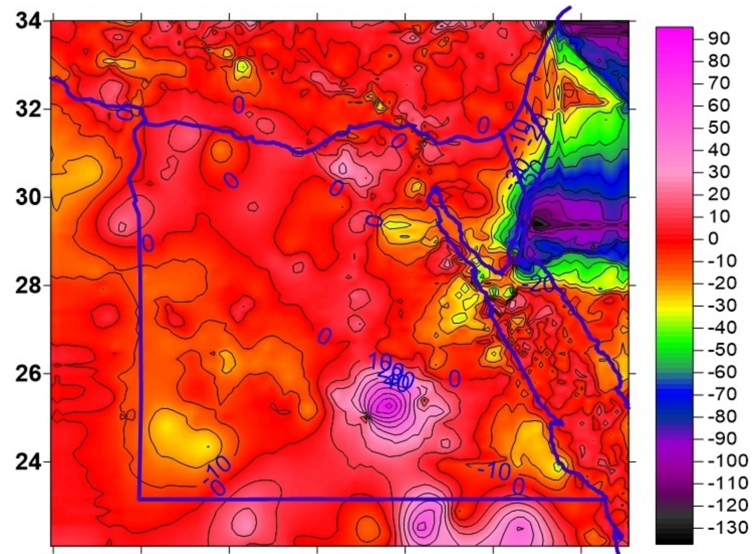

Fig.5 Δg_{fa} residuals referred to EGM GOCO05c with terrestrial data for the whole area of Egypt.

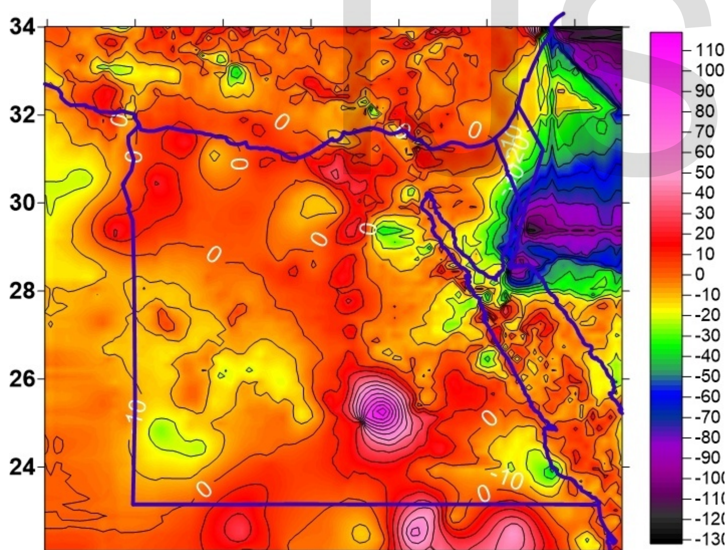

Fig.4 Δg_{fa} residuals referred to EGM NULP-02swith terrestrial data for the whole area of Egypt.

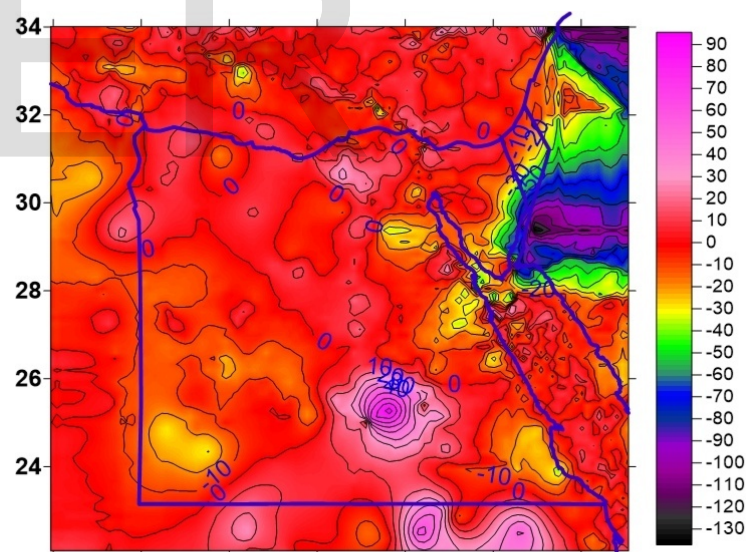

Fig.6 ,Δg_{fa} residuals referred to EGM GOCO with terrestrial data for the whole area of Egypt.

TABLE 6

GEOID COMPUTATION FROM DIFFERENT EGMS OVER WHOLE EGYPT TERRITORY OVER GRID POINTS [1'X1']

Geoid undulation	Min m	Max m	Mean m	R.M.S. m	STD. DEV.Of the residual m
$N_{XGM2016}$	1.279	27.784	14.874	15.317	3.656
$N_{NULP-02s}$	1.228	27.060	-14.144	15.302	3.645
$N_{GOCO05c}$	1.277	27.716	14.875	15.316	3.655
N_{GOCO}	1.286	27.771	14.866	15.309	3.658

TABLE 7

ENHANCED XGM2016 AFTER COMPUTATIONS FROM THE SECOND STEP OVER WHOLE EGYPT TERRITORY OVER GRID POINTS [1'X1']

Geoid undulation	Min m	Max m	Mean m	R.M.S. m	STD. DEV.Of the residual m
$N_{Enhanced\ XGM2016}$	0.2331	27.745	14.486	14.920	3.569

TABLE 8

THE INTERNAL ACCURACY TEST FOR ENHANCED XGM2016 AFTER COMPUTATIONS FROM THE SECOND STEP OVER WHOLE EGYPT TERRITORY GRID POINTS [1'X1']

Geoid undulation	Min m	Max m	Mean m	R.M.S. m	STD. DEV.Of the residual m
N_{obs} - $N_{Enhanced\ XGM2016}$	-0.313	0.715	0.400	0.441	0.189

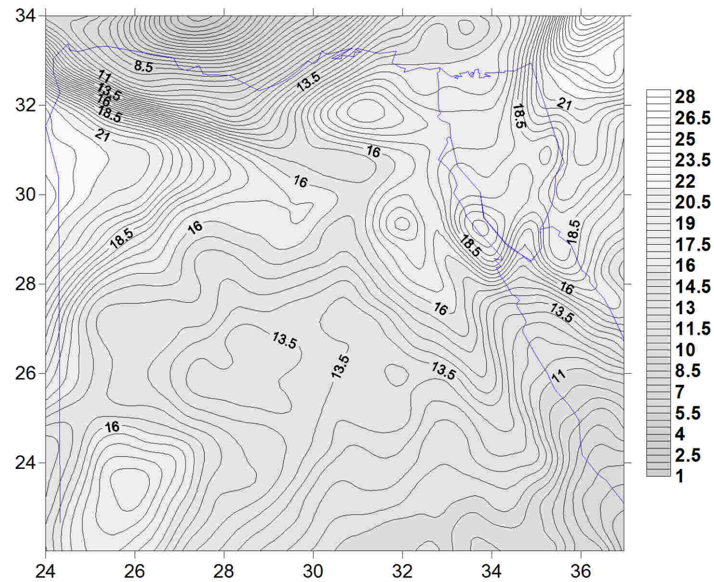

Fig.7, represents the geoid from EGM XGM2016 for the whole area of Egypt

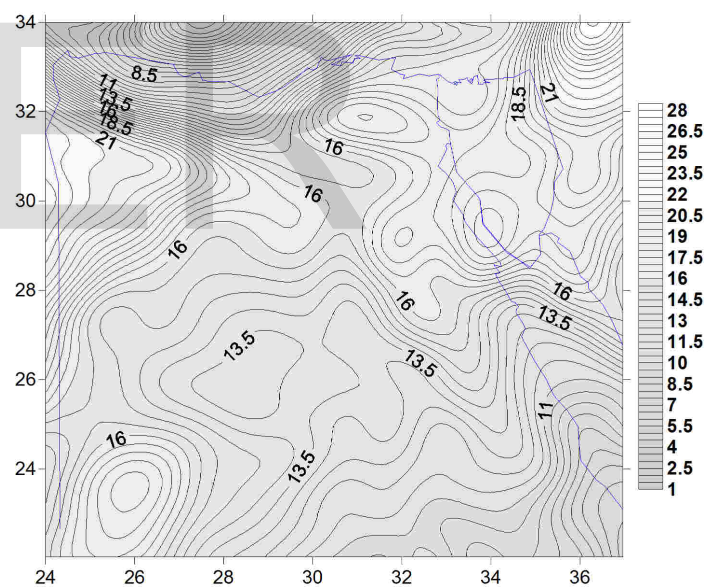

Fig.8 , represents the geoid from EGM NULP-02 for the whole area of Egypt

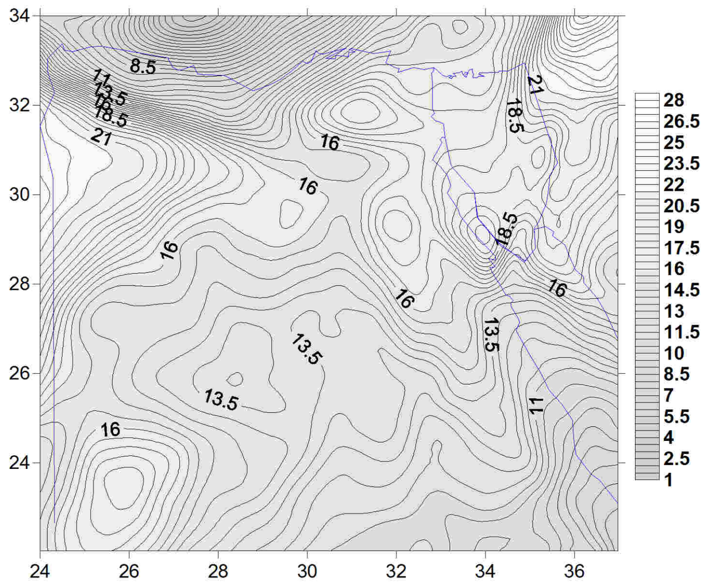

Fig.9 , represents the geoid from EGM GOCE 05c for the whole area of Egypt

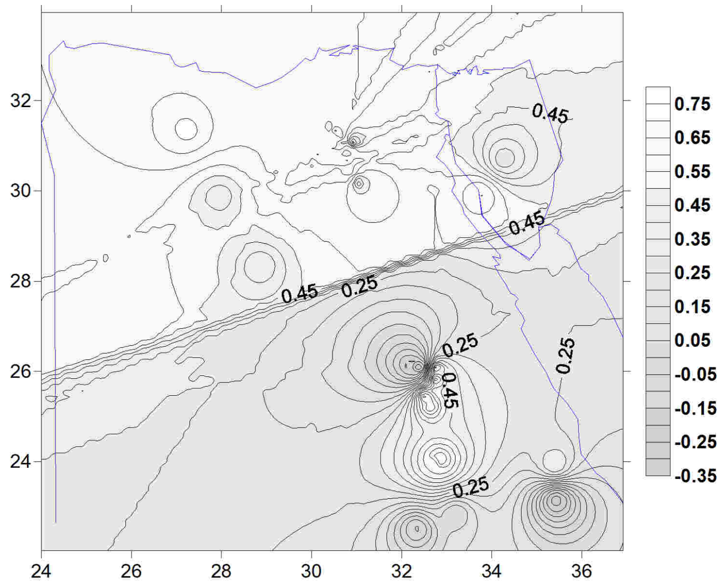

Fig.11, shown the Residual N terrestrial - N XGM2016 for the whole area of Egypt with contour interval 0.050 m

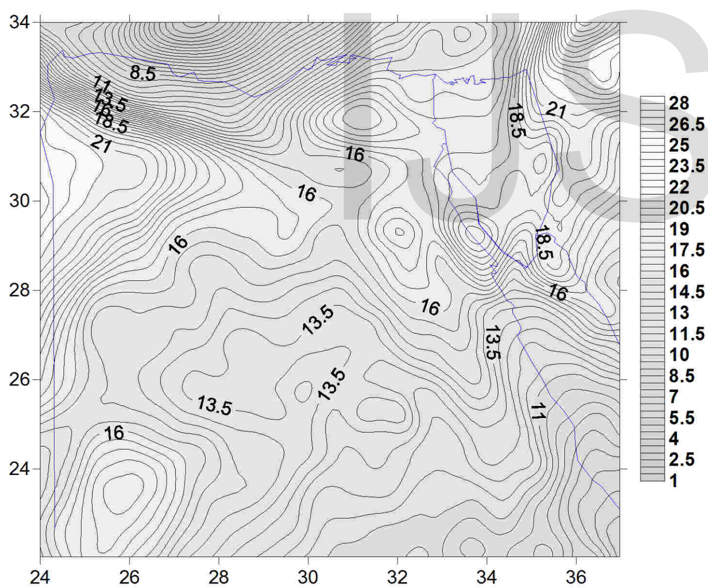

Fig.10 represents the geoid from EGM GOCE for the whole area of Egypt with 0.50 m contour interval

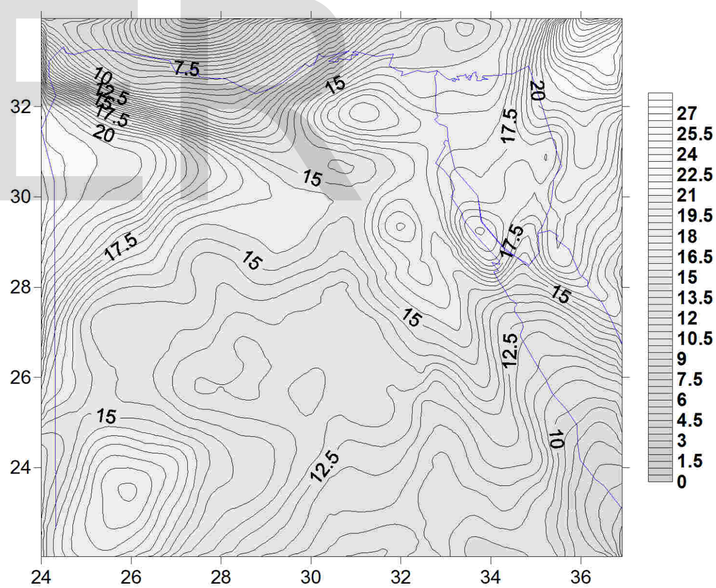

Fig.12 represents an enhanced EGM XGM2016 geoid from for the whole area of Egypt

Figure [3 to 6] describes the residuals between terrestrial free air gravity anomaly and those anomalies derived from XGM2016, NULP-02s, GOCO05c and GOCO harmonic models respectively over a grid [1'x1'] for whole Egypt territory.

Figure [7 to 10] represents geoid of the Different geopotential models XGM2016, NULP-02s, GOCO05c and GOCO, harmonic models, respectively.

While figure[11] illustrate the Residual [N terrsterial - N XGM2016] for the whole area of Egypt, residuals are not smoothing as expected, relevant to the impact of lake terrestrial data and heterogeneous distributions over the whole area.

Unfortunately, the performance of EGM 2008 which is considered as the main part of EGM GOCO with high spherical harmonic coefficient up to 2190 has imperfect performance all over EGYPT as it was expected, while the accepted accuracy from this model mostly because GOCE data included into it. The weak performance of this model comparing to the others due to wrong terrestrial data which are included in EGM 2008 model (Abd-Elmotaal, 2008), especially with GPS\leveling points data were most of them are wrong but for the free air gravity anomaly values they vary from region to another (Amin et al., 2013) within Egypt territory.

5. Conclusion

After assessment to determine which EGM choosing for enhancing utilizing the available GPS/ leveling scattering data points. The four EGMs under investigations have from the results a good performance over EGYPT and areas extends by approximate two degrees from each side, but the model XGM2016 has more accuracy than the other model as seen in table [5], [6], and from the results shown in figure [3].

The assessment results of the comparison among the different geopotential model with the land gravity data "free air gravity anomalies and GPS\Leveling discrete points" under investigation in this study have indicated the outstanding performance of EGM [XGM2016] to the other examined GGMs.

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An enhancement of EGM [XGM2016] with the GPS\Leveling discrete data points clarify the model is enhanced by 2.6% than the original one, if there is enough terrestrial data over the whole territory this ratio defiantly will be increase.

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6. References

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